The Duchess Reel

A 6 x 32 bar reel for 3 couples in a circle.
Composed 12 June 2012 by Stanford Ceili.
Modified from Duke Reel (traditional).

Assume circle is ordered, counter-clockwise: Man-1 (M1), Lady-1 (L1); Man-2 (M2), Lady-2 (L2); and Man-3 (M3), Lady-3 (L3).

(8) **Ring Right & Left.**

(16) **Slipsides.**
- (8) Slipsides with partner.
- (8) Slipsides with corner.

(8) **Link Arms (Uillinn in Uillinn).**
- (2) Turn partner by Right Arm.
- (2) Turn corner by Left Arm.
- (4) Repeat. End facing partner.

(8) **1st Interlace.**
Grand Chain but without hands. End past partner, progressed, with lady to Left of original partner and man to Right of original partner.

(8) **Ring Right & Left.**

(8) **Mirrored Interlace.**
M1 leads current partner (L2, not L1) to begin an interlace with couple on his Left (M3, L1). Simultaneously, M1 interlaces with couple on L2's Right (M2, L3). End with M1 and L2 on opposite side (between L3, M3). Circle is now ordered (counter-clockwise): M2, L3; M1, L2; and M3, L1.

(32) **Bend The Ring & Circle.**

(8) **Around The House.**

(96) **Repeat.**
Regardless of which couple dances the Mirrored Interlace, all couples will end with original partners.

Caller's Notes for The Duchess Reel:

(8) **Ring Right & Left.**

(16) **Slipsides.**

(8) **Link Arms (Uillinn in Uillinn).**

(8) **1st Interlace.**

(8) **Ring Right & Left.**

(8) **Mirrored Interlace.**

(32) **Bend The Ring & Circle.**

(8) **Around The House.**

(96) **Repeat.**

Note: after 2 repetitions, all dancers have returned to the same position in the circle (although the circle may have rotated, which may cause dancers to believe that progression is still ongoing). For more details, see the analysis below, by Stanford Ceili member Yoyo Zhou.

Dance Analysis

by Yoyo Zhou
2012 June 13

Here's a bit of group theory for our variation on the "Duke Reel." If you were wondering how we have a set of 3 couples, swap around symmetrically, and come back to our original places after *2* (not 3) iterations . . . .

We started with 3 couples in a ring (they are 1-2, 3-4, 5-6); pretend the line below wraps around:

1 2 3 4 5 6

Then in the first interlace, we progressed past our partner: 2 1 4 3 6 5, or equivalently

1 4 3 6 5 2

Now the couples are 1-4, 3-6, 5-2.

Following this, what we did was: one couple (it's a little unclear who) splits the other two and eventually ends up on the other side. Supposing 5-2 do this, we end up as

1 4 5 2 3 6

But actually, we'll end up this way regardless of which couple does the second interlace.

Now what happened? It looks like 3 and 5 traded places, and 2 and 4 traded places. If we run through the dance again, they'll just trade back! But how?

We can make a few simplifications:

- Between each figure (where we record dancers' positions), we always have 3 couples in a ring, with men on the left and ladies on the right.
- We're only interested in who ends up partnered with whom, so we can leave somebody in a fixed position in the ring (say, dancer #1) and take it from their perspective.

From #1's perspective, the other 2 men can be in either of 2 places, and independently, the 3 ladies can be in any of 3 places. So our group is equivalent to $S_2 \times S_3$, the product of symmetric groups.[1]

One set of generators of the group is

a: rotate the ladies

b: swap the ladies in the other 2 couples

c: swap the men in the other 2 couples

$aaa = 1$ (the identity); $bb = 1$; $cc = 1$; a and c commute, b and c commute, but $abab = 1$;[2] and any transformation can be written as a combination of a, b, and c.

[1] http://mathworld.wolfram.com/SymmetricGroup.html explains this well for S_3. The other group S_2 has only 2 elements and is the finite simple group of order 2; for a wonderful non-explanation, see http://www.youtube.com/watch?v=UTby_e4-Rhg

[2] One way to convince yourself of this is to think of a as a rotation and b as a reflection.

Let's see what we danced:

The first interlace progression (pass your partner) is a. The supposed second interlace progression (move one couple to the other side) is bc. Altogether, we have abc.[3]

Doing it twice, we have $abcabc$. Since c commutes with the other two, this is the same as $ababcc = 1$. So abc has order[4] 2, and after 2 iterations, we're back where we started!

What else can we learn from math? Well, we can find the order of any element of the group (like a, bc, or $aabc$). So:

- If we want a dance that progresses all the way back to where we started after 3 iterations, we should do just a (or aa) each time.
- If we want a dance that only comes back after 6, we could do both a and c each time.
- But there are no elements of order 12, even though there are $2! * 3! = 12$ possible rearrangements.
- It's kind of hard to figure out how to do this on the fly and get the results you wanted.

The Duke Reel (mixer version) was being taught by Ammy Woodbury to the Intermediate class. Due to confusion over how the Second Interlace worked, causing the rest of the dance to not flow well, this variant was created.

[3] From earlier, "3 and 5 trade places" is c, while "2 and 4 trade places" is ab.
[4] The number of times it needs to be applied in succession to get back to the identity.